Kentucky Academic Standards for Mathematics: Conceptual Category Statistics and Probability

Statistics and Probability Overview

Interpreting Categorical and Quantitative Data	Making Inferences and Justifying Conclusions	Conditional Probability and the Rules of Probability	Using Probability to Make Decisions
• Summarize, represent and interpret data on a single count or measurement variable. • Summarize, represent and interpret data on two categorical and quantitative variables. • Interpret linear models.	• Understand and evaluate random processes underlying statistical experiments. • Make inferences and justify conclusions from sample surveys, experiments and observational studies.	• Understand independence and conditional probability and use them to interpret data. • Use the rules of probability to compute probabilities of compound events in a uniform probability model.	• Calculate expected values and use them to solve problems. • Use probability to evaluate outcomes of decisions.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Plus (+) Standards: Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

Statistics and Probability-Interpreting Categorical and Quantitative Data	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Summarize, represent and interpret data on a single count or measurement variable.	
Standards	**Clarifications**
KY.HS.SP.1 Represent the distribution of data with plots on the real number line (stem plots, dot plots, histograms and box plots). **MP.4, MP.5**	Students create appropriate graphical representations to compare differences in the shape, center, spread and presence of outliers and other unusual features of comparable data sets.
KY.HS.SP.2 Use statistics appropriate to the shape of the numerical data distribution to compare center (median, mean) and spread (interquartile range when comparing medians and standard deviation when comparing means) of different data distributions. **MP.2, MP.6**	Students use raw data and data from appropriate graphical representations to compare differences in the shape, center, spread and presence of outliers and other unusual features of comparable data sets.
KY.HS.SP.3 Interpret differences in shape, center and spread in the context of the distributions of the numerical data, accounting for the presence and possible effects of extreme data points (outliers). **MP.1, MP.7**	Students analyze contextual situations as they interpret differences in the shape, center, spread and presence of outliers and other unusual features of comparable data sets.
KY.HS.SP.4 (+) When appropriate, fit a normal distribution to a numerical data set for given mean and standard deviation and then estimate population percentages using the Empirical Rule and recognize that there are data sets for which such a procedure is not appropriate. **MP.1, MP.3**	Students use the empirical rule (68%-95%-99.7% rule), calculators and/or tables to estimate areas under the normal curve, recognizing when data sets are skewed this can be problematic.
Attending to the Standards for Mathematical Practice	
Students use technology to visualize data using stem plots, dot plots, histograms and box plots ( ). After the data have been collected, students are precise about choosing the appropriate analyses and representations to reveal the variability in the data ( ). Students analyze quantitative data and classify any observation(s) that deviate(s) considerably from the majority of data within a distribution as potential outliers ( ).	

The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.

Statistics and Probability- Interpreting Categorical and Quantitative Data	
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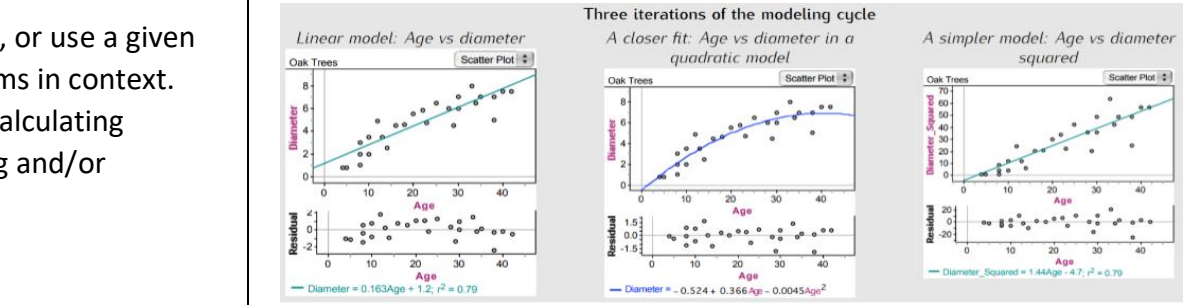
Cluster: **Summarize**, **represent and interpret data on two categorical and quantitative variables.**	
Standards	**Clarifications**
KY.HS.SP.5 Summarize categorical data for two or more categories in frequency tables. Calculate and interpret joint, marginal and conditional relative frequencies (probabilities) in the context of the data, recognizing possible associations and trends in the data. **MP.2**, **MP.7**	Students use frequency tables to both calculate probabilities, as well as determine relationships between the variables represented in those tables.
KY.HS.SP.6 Represent data on two quantitative variables on a scatter plot and describe how the explanatory and response variables are related. a. Calculate an appropriate mathematical model, or use a given mathematical model, for data to solve problems in context. b. Informally assess the fit of a model (through calculating correlation for linear data, plotting, calculating and/or analyzing residuals). **MP.3**, **MP.4**, **MP.5**	Emphasize linear, quadratic and exponential models as illustrated below. 
Attending to the Standards for Mathematical Practice	
Students discover structures or patterns in data to answer statistical questions using tables or appropriate representations ( ). Students informally determine whether a selected model is appropriate for a set of data and use technology when appropriate to do so ( ). Students draw and discuss conclusions about a statistical question ( ) using appropriate mathematical models.	

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Statistics and Probability-Interpreting Categorical and Quantitative Data	
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Cluster: Interpret linear models.

Standards	Clarifications
KY.HS.SP.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **MP.1**, **MP.2**	Students demonstrate interpreting slope in the context of a given situation when examining two variable statistics as being "for each additional known unit increase in an explanatory variable, we expect or predict a known unit increase (or decrease) in the response variable." Students demonstrate interpreting intercept in the context of a given situation when examining two variable statistics as being "the predicted known unit of a response variable when the explanatory variable is zero known units."
KY.HS.SP.8 Understand the role and purpose of correlation in linear regression. a. Use technology to compute correlation coefficient of a linear fit. b. Interpret the meaning of the correlation within the context of the data. c. Describe the limitations of correlation when establishing causation. **MP.5**, **MP.6**	a. Students use technology to perform the calculation of: $r = \frac{\Sigma(x-\overline{x})}{\sqrt{\Sigma(x-\overline{x})^2}\sqrt{\Sigma(y-\overline{y})^2}}$ b. Students understand correlation measures linear associations between two quantitative variables addressing the direction (positive or negative) and the relative strength of the given association. c. Students understand one of the most common misinterpretations of correlation is to think of it as a synonym for causation. A high correlation between two variables (suggesting a statistical association between the two) does not imply one causes the other.

Attending to the Standards for Mathematical Practice

Students interpret the results to a statistical question and relate the results to the context of the data (,). Students use technology to compute correlation coefficients ( ). Students recognize that correlation is an indication of a linear relationship between two quantitative variables and not simply another word for association ( ).

Statistics and Probability-Making Inferences and Justifying Conclusions	
Standards for Mathematical Practice	
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Cluster: Understand and evaluate random processes underlying statistical experiments.	
Standards	**Clarifications**
KY.HS.SP.9 Understand statistics as a process for making inferences and justifying conclusions about population parameters based on a random sample from that population. **MP.1**, **MP.3**	Students use sample statistics (mean and standard deviation) obtained from random samples to help estimate population parameters which are unknown values.
KY.HS.SP.10 Decide if a specified model is consistent with the results from a simulation. **MP.3**, **MP.6**	If a model shows a spinning coin falls heads-up with probability of 0.5, would a result of 5 tails in a row cause you to question the model?
Attending to the Standards for Mathematical Practice	
Students follow the progression of the statistical problem-solving process to investigate answers to a statistical question ( ). Students justify their conclusions, communicate them to others (orally and in writing) and critique the conclusions of others ( ). Students are precise about choosing the appropriate analyses and representations that account for the variability in the data ( ).	

Statistics and Probability- Making Inferences and Justifying Conclusions	
Standards for Mathematical Practice	
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Cluster: Make inferences and justify conclusions from sample surveys, experiments and observational studies.	
Standards	**Clarifications**
KY.HS.SP.11 Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each. **MP.3**, **MP.8**	Students understand a random selection of 100 students from your school will allow you to draw some conclusions about all the students in the school, whereas taking your class as a sample will not allow that generalization. Students recognize experiments involve imposing treatments on units/subjects, whereas observational studies do not.
KY.HS.SP.12 Use data from a sample survey to estimate a population mean or proportion and explain how bias may be involved in the process. **MP.4**, **MP.7**	KY.HS.SP.12 differs from KY.HS.SP.9 in that results from non-random samples (Voluntary Response and Convenience) generate biased results when compared with more appropriate, random samples of the same population.
KY.HS.SP.13 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between estimates or statistics are significant. **MP.3**, **MP.8**	Hypotheses can be tested to determine if significant differences between two treatments exist using statistical data. If significance exists, claims and conclusions can be made about the treatment.
Attending to the Standards for Mathematical Practice	
Students compare and contrast the different roles randomization plays in data collection ( ). Students look for patterns in the variability around the structure ( ).	

Statistics and Probability-Conditional Probability and the Rules of Probability	
Standards for Mathematical Practice	
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics.	MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.
Cluster: Understand independence and conditional probability and use them to interpret data.	
Standards	**Clarifications**
KY.HS.SP.14 Describe events as subsets of a sample space. Use characteristics (or categories) of the outcomes, such as, • as unions, "A or B," that are mutually exclusive events and • as unions, "A or B," that are non-mutually exclusive events and • as intersections, "A and B," and • as complements of other events, "not A." to calculate basic probabilities. **MP.1**, **MP.2**	A union of two events, "A or B," includes *all* elements in both events notated by: $A \cup B$. Addition Rule for mutually exclusive events: If A and B are mutually exclusive, P(A or B) = P(A) + P(B). Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B) and interpret the answer in terms of the model. An intersection, "A and B," of two events includes all overlapping elements notated by: $A \cap B$. A complement for any event A, P(not A) = 1 – P(A).
KY.HS.SP.15 Understand the concept of independence. a. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their individual probabilities, P(A) x P(B) b. (+) Determine whether two events are independent and provide a justification to support the decision. c. Recognize and explain the concept of independence in everyday language and everyday situations. **MP.1**, **MP.6**	a. Events *A* and *B* are independent if and only if P(*A* and *B*) = *P*(*A*)*P*(*B*).
KY.HS.SP.16 Understand the concept of conditional probability. a. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*). b. (+) Interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A* and the conditional probability of *B* given *A* is the same as the probability of *B*.	a. For any two events *A* and *B,* P(*A* given *B*) = $\frac{P(A \text{ and } B)}{P(A)}$.

Standards	Clarifications
c. Recognize and explain the concept of conditional probability in everyday language and everyday situations. d. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A and* interpret the answer in terms of the model. **MP.1**, **MP.3**	
KY.HS.SP.17 (+) Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide whether events are independent and to approximate conditional probabilities. **MP.2**, **MP.4**	Students collect their own data or use data obtained from a random sample of their choosing and construct two-way frequency tables from their sample in order to determine independence and calculate probabilities.

Attending to the Standards for Mathematical Practice
Students encounter chance events in real contexts, including situations involving both dependent and independent events, are able to determine the difference between the contexts and fluently select and use appropriate formulas ( ). Students consider whether the occurrence of one event affects the probability of the other event ( ) in order to determine if two events are independent. Students analyze and discuss a variety of sources such as contingency tables to provide a context for conditional probability ( ). Students consider how conditions or assumptions affect the computation of a probability ( ).

Statistics and Probability-Conditional Probability and the Rules of Probability	
Standards for Mathematical Practice	
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Cluster: Use the rules of probability to compute probabilities of compound events

Standards	Clarifications
KY.HS.SP.18 (+) Apply the General Multiplication Rule, P(A and B) = P(A)P(B|A) = P(B)P(A|B), in a uniform probability model and interpret the answer in terms of the model. **MP.1**, **MP.2**	Consider an experiment where two cards are drawn without replacement. Define events A and B: A = 1st card drawn is a king B = 2nd card drawn is a king P(B|A) is the probability that the second card is a king given the first card drawn was a king. In that case, there will be 3 kings left out of 51 cards, so P(B|A) = 3/51 P(A and B) = P(1st is a king and 2nd is a king) P(A and B) = P(1st king) • P(2nd is a king, given 1st is a king) P(A and B) = (4/52) (3/51) P(A and B) = P(A) • P(B|A)
KY.HS.SP.19 Use permutations and combinations to compute probabilities. a. Distinguish between situations that can be modeled using counting techniques, including Fundamental Counting Principle, permutations and combinations. b. Perform calculations using the appropriate counting technique, including simple probabilities. c. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. **MP.1**, **MP.8**	Permutations are calculated when order matters. Combinations are calculated when order does not matter. Number of permutations of *n* items taken *r* at a time: ${}_nP_r = \frac{n!}{(n-r)!}$ Number of combinations of *n* items taken *r* at a time: ${}_nC_r = \frac{n!}{(n-r)!r!}$

Attending to the Standards for Mathematical Practice
Students recognize and solve real-world problems using the Fundamental Counting Principle, Permutations and Combinations ( ). Students identify patterns to generalize a formula for calculating permutations and combinations ( ).

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Statistics and Probability-Using Probability to Make Decisions	
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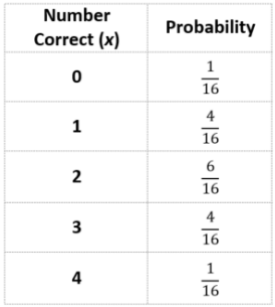
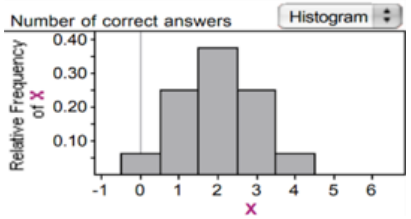
Cluster: Calculate expected values and use them to solve problems.

Standards	Clarifications
KY.HS.SP.20 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same appropriate graphical displays as for data distributions. **MP.3**, **MP.6**	Students realize random variables are different from the variables used in other high school domains and random variables are functions of the outcomes of a random process and thus have probabilities attached to their possible values. A possible example of a probability distribution:  **Number Correct (x) / Probability** 0 — $\frac{1}{16}$ 1 — $\frac{4}{16}$ 2 — $\frac{6}{16}$ 3 — $\frac{4}{16}$ 4 — $\frac{1}{16}$ 
KY.HS.SP.21 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution and use the value in analyzing decisions. **MP.1**, **MP.8**	The expected value/mean of a discrete random variable is $\mu = E(x) = \Sigma xp(x)$.
KY.HS.SP.22 (+) Develop a probability distribution for a random variable. a. Find an expected value based on a sample space in which theoretical probabilities can be calculated. b. Find an expected value based on a sample space in which empirical probabilities can be calculated. **MP.2**, **MP.8**	a. (+) Theoretical probability is given by the number of ways a particular event can occur divided by the total number of possible outcomes. b. (+) The empirical probability of an event is given by number of times an event occurs divided by the total number of incidents observed.

Statistics and Probability-Using Probability to Make Decisions	
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Cluster: Use probability to evaluate outcomes of decisions.	
Standards	**Clarifications**
KY.HS.SP.23 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. b. Evaluate and compare strategies on the basis of expected values. c. Use calculated expected values to make fair decisions and formulate strategies. **MP.6**, **MP.8**	Students use expected values to play a role in decision making in a variety of contexts.

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